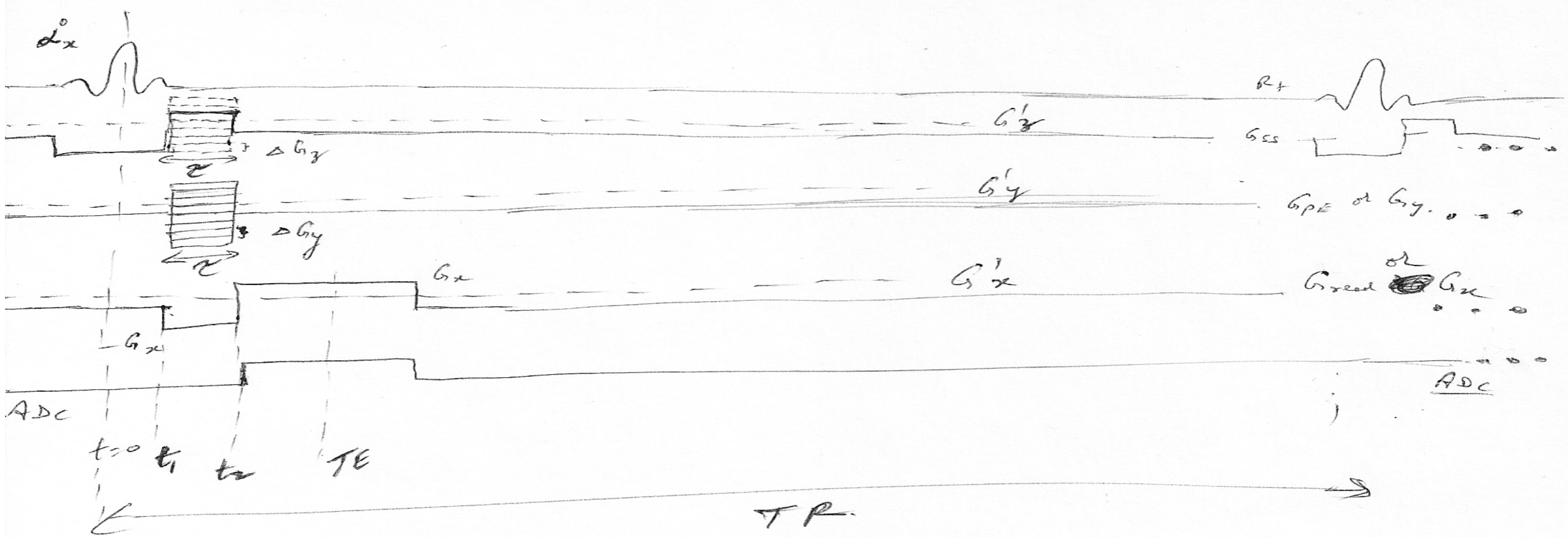
- T2’
- Phase Images in GRE
- Tissue Magnetic Susceptibility Affects.



$$S(t') = \int dx \int dy \int dz \, \rho(x,y,z)\, e^{-j\phi(x,y,z,t')}$$

$$\begin{aligned}\phi(x,y,z,t') &= \gamma(m\Delta G_y \tau)\, y + \gamma(n \Delta G_z \tau)\, z \\ &\quad + \gamma G_x t' \left(x + \frac{G'_x x}{G_x} + \frac{G'_y y}{G_x} + \frac{G'_z z}{G_x} \right) \\ &\quad + \gamma TE \left(G'_x x + G'_y y + G'_z z \right)\end{aligned}$$

$$\equiv \gamma TE \left(\frac{\partial \Delta B(x,y,z)}{\partial x} x + \frac{\partial \Delta B(x,y,z)}{\partial y} y + \frac{\partial \Delta B(x,y,z)}{\partial z} z \right)$$

$$\equiv \gamma TE \left(\Delta B(x,y,z) \right)$$

$$\phi(x,y,z,t') = \gamma(m\,\Delta G_y\,\tau)\,y + \gamma(n\,\Delta G_z\,\tau)\,z$$

$$+\ \gamma G_x t' \left(x + \frac{G_x' x}{G_x} + \frac{G_y' y}{G_x} + \frac{G_z' z}{G_x} \right)$$

$$\underline{+\ \gamma\,\Delta B\,TE} \equiv \phi_{gre}$$

additional phase term in gradient echo sequence.

$$\tilde{\rho}(\gamma, TE) = d^3r\, \rho(\vec{r})e^{-i\gamma B(\vec{r})T_E}$$

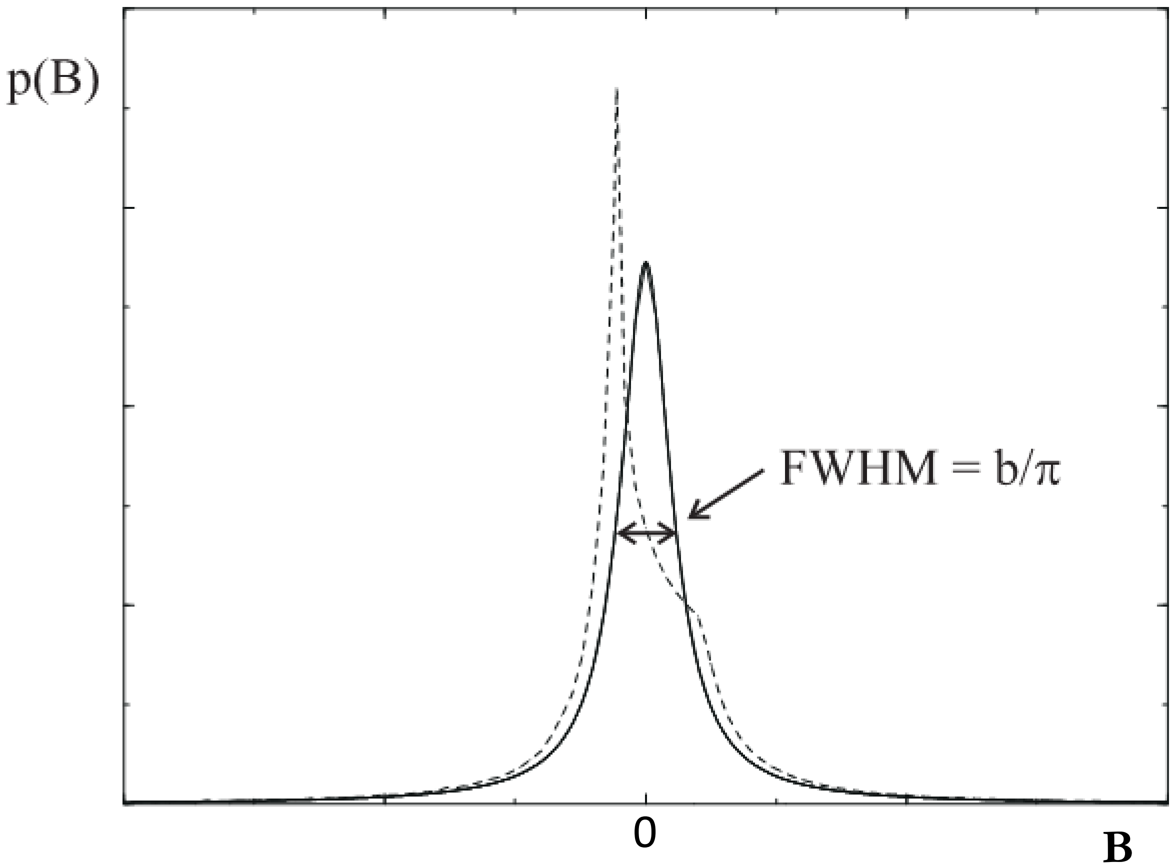
$$= \int_V d^3r\, \rho(\vec{r}) \int dB\, \delta(B(\vec{r}) - B)e^{-i\gamma BT_E}$$

$$= \frac{\rho_0}{V} \int d(\Delta B) \int d^3r\, \delta(\Delta B(r) - \Delta B)\, e^{-j \cdot \gamma \cdot \Delta B \cdot TE}$$

$$P(B) = \frac{1}{\rho_0 V} \int \rho(\vec{r})\delta(B(\vec{r}) - B)\, d^3r$$

$$\Delta B(r, \theta) = \frac{1}{3r^3} \Delta\chi B_0 R^3 (3\cos^2\theta - 1)$$

For a spherical inclusions with susceptibility difference of $\Delta\chi$



S(TE)=So exp(-γbTE)

b= π .2|ΔB|

$$\frac{1}{T_2^*} = \frac{1}{T_2} + \gamma|\Delta B|$$

$$\eta = \frac{NV_\mu}{V_0}$$

$$\begin{aligned}\hat{\rho}(T_E) &= \rho_0(1-\eta-\eta g(\omega_d T_E)) \\ &= \rho_0(1-\eta(1+g(\omega_d T_E)))\end{aligned}$$

$$g(\omega_d T_E) = \begin{cases} \frac{2}{5}(\omega_d T_E)^2 & \omega_d T_E \ll 1 \\ a_1\omega_d(T_E - t_s) + i a_2 \omega_d T_E & \omega_d T_E \gg 1 \end{cases}$$

$$\begin{aligned} a_1 &= \frac{2\pi}{3\sqrt{3}} \simeq 1.21 \\ a_2 &= \frac{2}{3}\left(\frac{1}{\sqrt{3}}\ln\frac{(\sqrt{3}+1)}{(\sqrt{3}-1)} - 1\right) \simeq -0.16 \end{aligned}$$

$$t_s = (a_1\omega_d)^{-1}$$

$$\begin{aligned}\omega_d &= \gamma\mu/R^3 \\ &= \frac{1}{3}\gamma\Delta\chi B_0\end{aligned}$$

$$\hat{\rho}(T_E) = \rho_0(1-\eta)e^{-R_2'(\eta,T_E)T_E}$$

$$R_2'(\eta, T_E) = \eta g(\omega_d T_E)/T_E$$

t_s is close to zero

$$\begin{aligned} R_2' &= \eta a_1 \omega_d \\ &= \frac{2\pi}{9\sqrt{3}}\eta\gamma\Delta\chi B_0 \end{aligned}$$

$$\frac{1}{T_2^*} = \frac{1}{T_2} + \kappa\gamma|\Delta B|$$

$$\mathbf{B} = \mu_0(\mathbf{H} + \mathbf{M})$$

μ_0 is the absolute permeability of free space

$$\mathbf{M} = \chi\mathbf{H} \qquad \mathbf{B} = \left(\frac{1+\chi}{\chi}\right)\mu_0\mathbf{M}$$

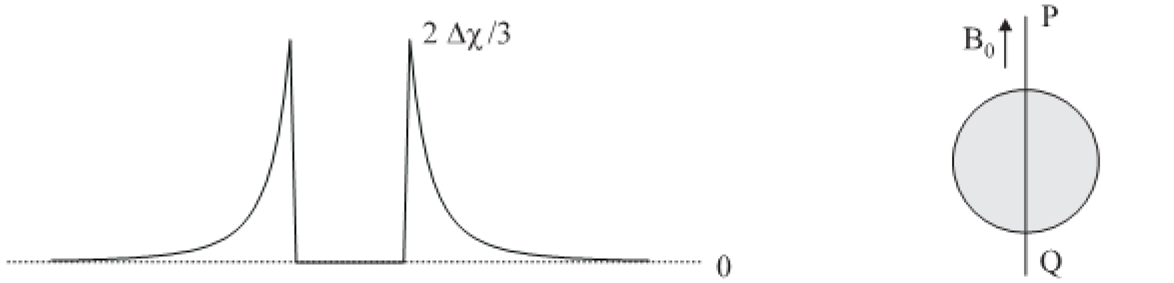
$$\mathbf{B}(\mathbf{r}) = \mathbf{B}_0 + \frac{\mu_0}{4\pi}\int_{V'} \mathrm{d}^3 r' \left\{ \frac{3M(\mathbf{r}') \cdot (\mathbf{r}-\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|^5}(\mathbf{r}-\mathbf{r}') - \frac{M(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|^3} \right\}$$

$$\mathbf{M}(\mathbf{r}) = \frac{\chi(\mathbf{r})}{\mu_0(1+\chi(\mathbf{r}))}\mathbf{B}(\mathbf{r})$$

TABLE 2.1 Magnetic Susceptibilities of a few Biological Tissue

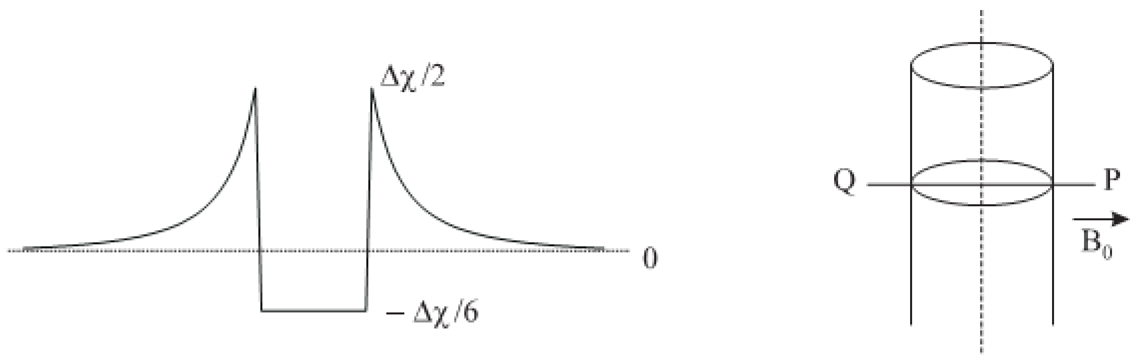
Tissue	Magnetic Susceptibility[a]
Lipids (stearic acid)	-10×10^{-6}
Cortical bone	-12.82×10^{-6}
Hemoglobin protein (without Fe ions)	-9.91×10^{-6}
Fully deoxygenated red blood cell	-6.56×10^{-6}
Fully deoxygenated whole blood (assuming Hct[b] = 0.45)	-7.9×10^{-6}
Ferritin (completely loaded with 4500 Fe^{3+} ions)	$+520 \times 10^{-6}$
Pure water	-9.05×10^{-6}

	Internal field shift = macro + Lorentz correction	External field shift = macro + Lorentz correction
sphere	$\frac{1}{3}\chi_e B_0$	$\frac{\Delta\chi}{3}\frac{a^3}{r^3}(3\cos^2\theta - 1)B_0 + \frac{1}{3}\chi_e B_0$
cylinder	$\frac{\Delta\chi}{6}(3\cos^2\theta - 1)B_0 + \frac{1}{3}\chi_e B_0$	$\frac{\Delta\chi}{2}\frac{a^2}{\rho^2}\sin^2\theta\cos 2\phi B_0 + \frac{1}{3}\chi_e B_0$



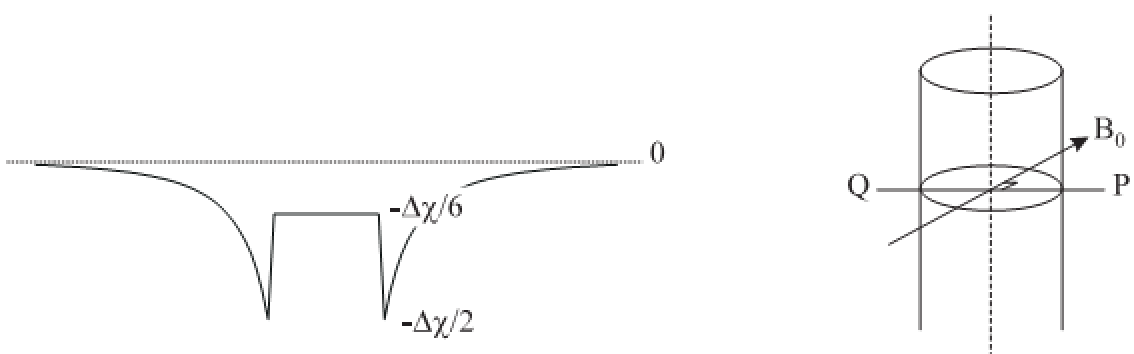
$2\ \Delta\chi/3$
0
B_0
P
Q

(a)



$\Delta\chi/2$
0
$-\ \Delta\chi/6$
Q
P
B_0

(b)

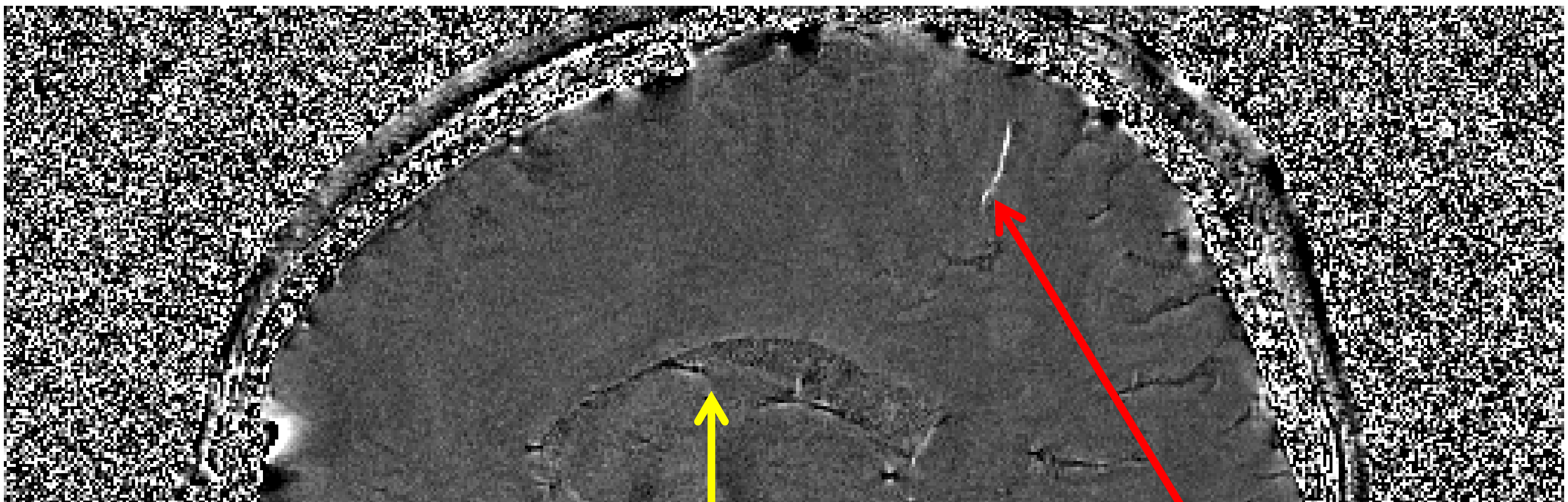


0
$-\Delta\chi/6$
$-\Delta\chi/2$
B_0
Q
P

Magnetic Susceptibility Field effects: Object shape and orientation dependency

Phase ($\varphi=\gamma\Delta B TE$) value inside the vein changes with the orientation of the vein.

$$\Delta B_{in}(r) = \frac{\chi B_0}{6} \cdot (3\cos^2\theta - 1)$$



Perpendicular to B_o

Parallel to B_o

Measuring ΔB using Double Echo GRE sequence

$$
\begin{aligned}
\phi(\vec{r}, T_{E_1}) &= \phi_0 - \gamma \Delta B(\vec{r}) T_{E_1} \\
\phi(\vec{r}, T_{E_2}) &= \phi_0 - \gamma \Delta B(\vec{r}) T_{E_2}
\end{aligned}
$$

$$
\Delta B(\vec{r}) = \frac{\phi(\vec{r}, T_{E_2}) - \phi(\vec{r}, T_{E_1})}{\gamma (T_{E_1} - T_{E_2})}
$$

$$
\phi_0 = \phi(\vec{r}, T_{E_1}) + \gamma \Delta B(\vec{r}) T_{E_1}
$$

Estimation of Oxygenation Levels

$$\chi_{blood} = \mathrm{Hct}\,(Y\chi_{oxy} + (1-Y)\chi_{deoxy}) + (1-\mathrm{Hct})\chi_{plasma}$$

$$\Delta\chi_{blood} = -\Delta Y(\chi_{deoxy} - \chi_{oxy})\mathrm{Hct}$$

$$\begin{aligned} \chi_{do} &\equiv \chi_{deoxy} - \chi_{oxy} \\ &= 4\pi \cdot (0.18)\ \text{ ppm per unit Hct} \end{aligned}$$

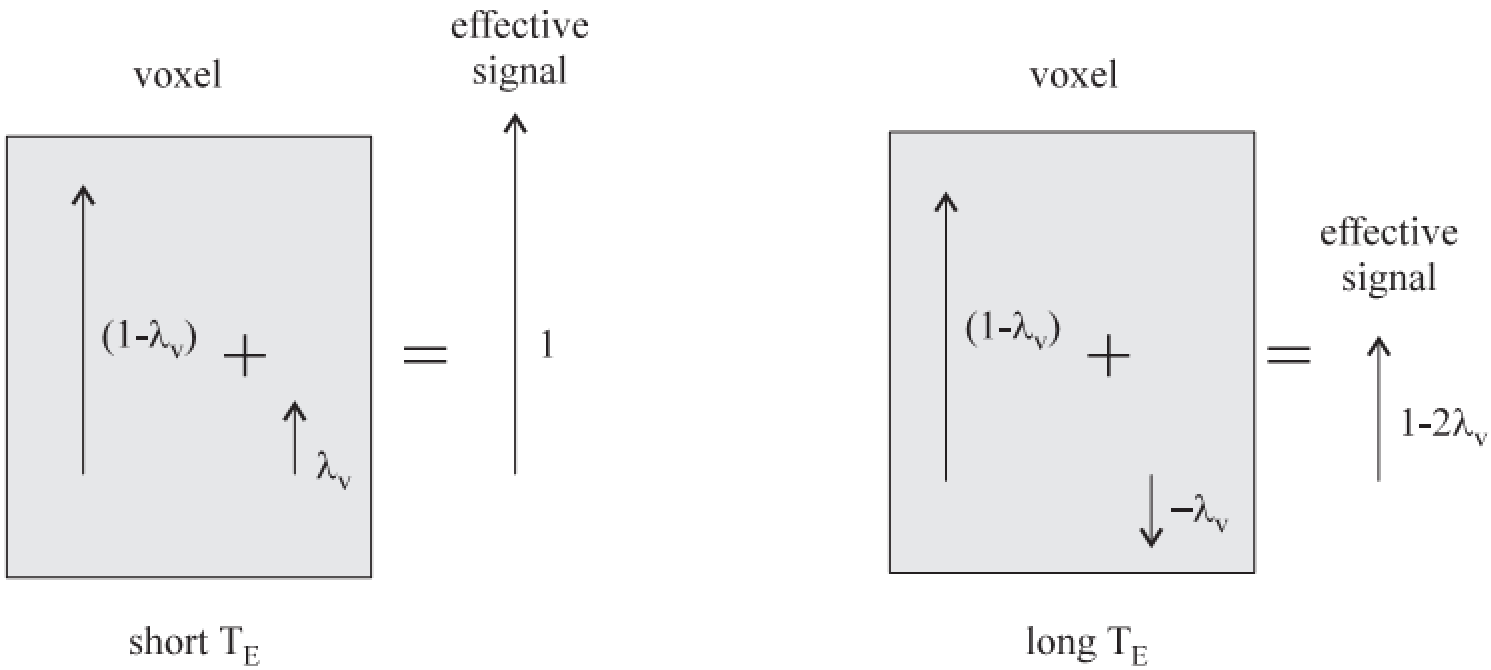
$$\chi_{blood,relative} \equiv \chi_{blood} - \chi_{surr} \simeq \chi_{blood} - \chi_{oxy} = 4\pi \cdot (0.18) \cdot \mathrm{Hct}(1-Y)\,\mathrm{ppm}$$

$$\begin{aligned} \phi_{blood} &= -\gamma\Delta B_{cyl,in}T_E \\ &= -\gamma\chi_{do}\mathrm{Hct}(1-Y)(3\cos^2\theta - 1)B_0T_E/6 \end{aligned}$$

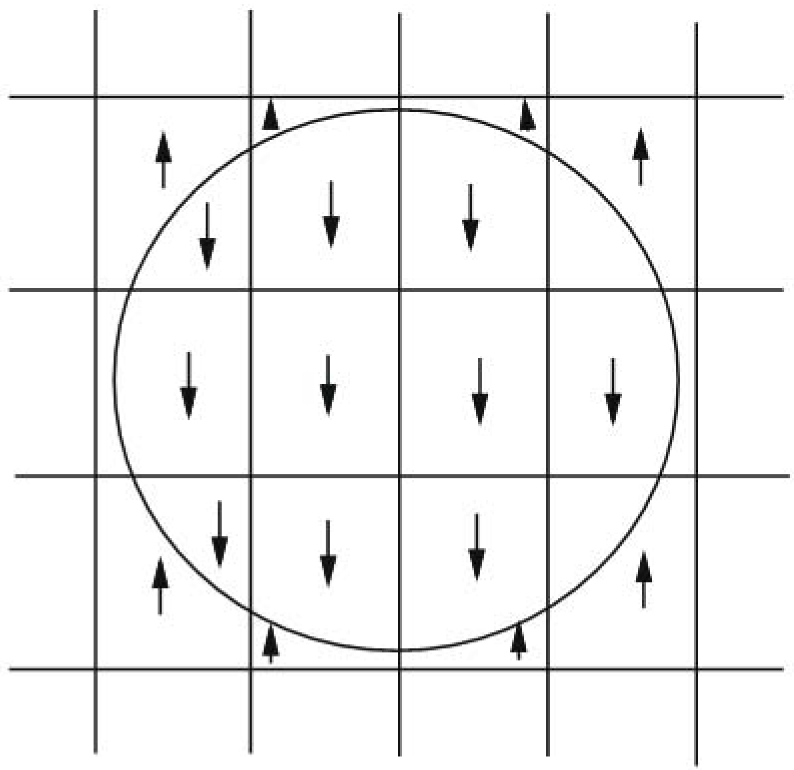
T_2 and T_2^* of Blood

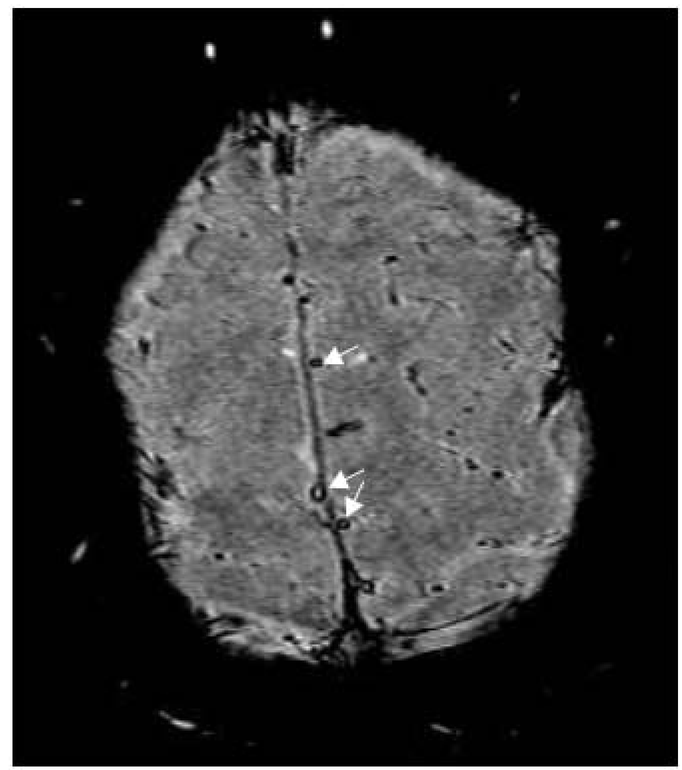
$$R_2 = R_{2,0} + \alpha(1 - Y) + \beta(1 - Y)^2$$

$$R_2^* = R_{2,0}^* + \alpha^*(1 - Y) + \beta^*(1 - Y)^2$$

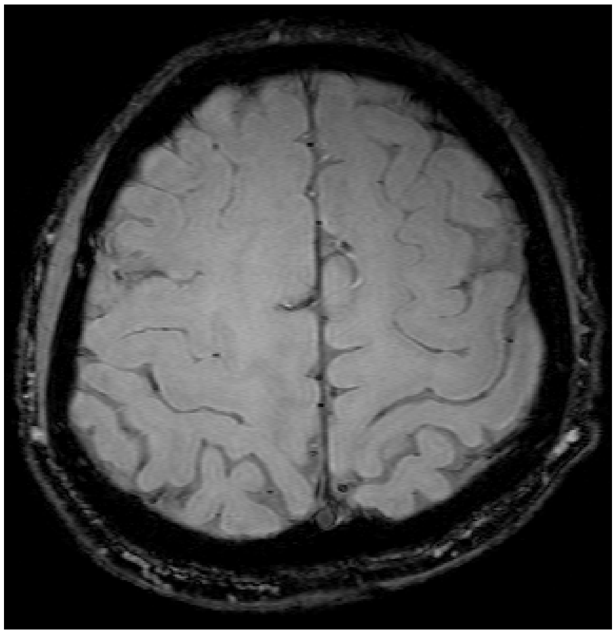


$$\hat{\rho}(T_E) = \rho_0 e^{-T_E/T_2^*}\left((1-\lambda_v) + \lambda_v e^{i\phi_v(T_E)}\right)$$

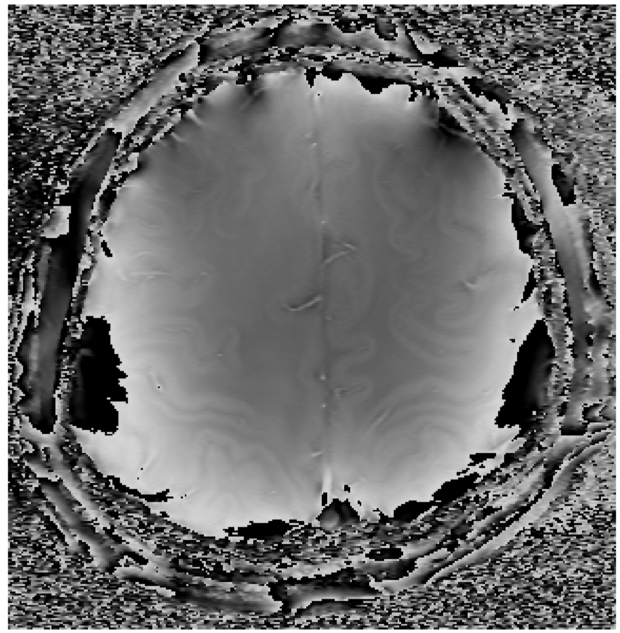




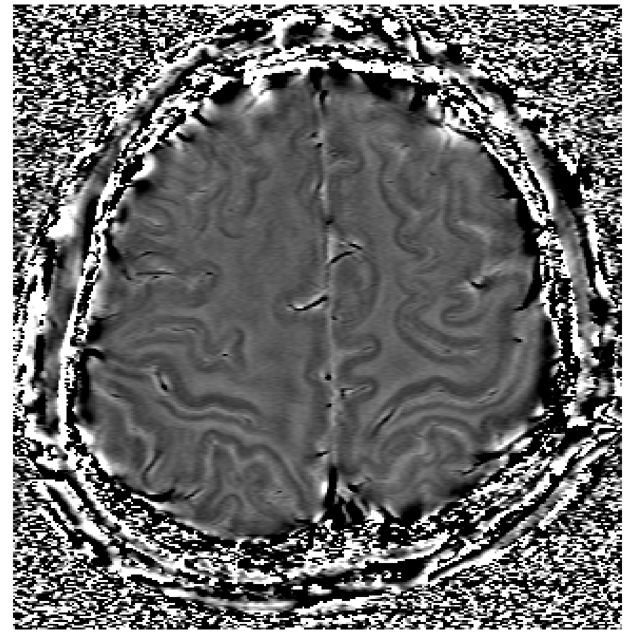
Susceptibility Weighted Imaging (SWI): Brief Introduction



Un processed Magnitude



Un-Filtered Phase

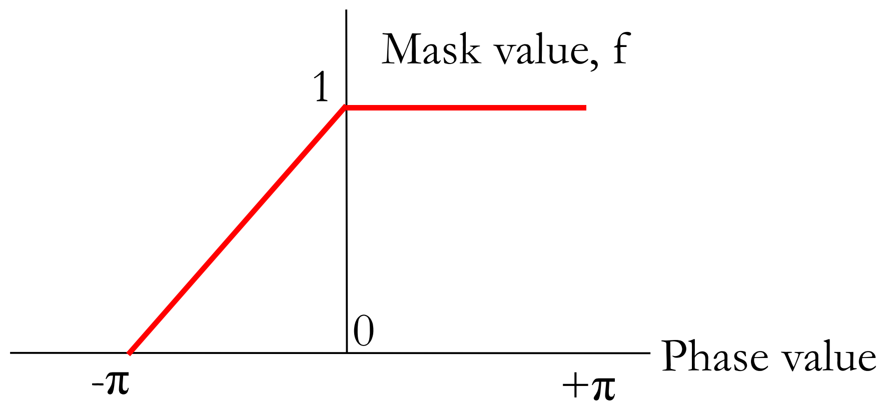


Filtered Phase

Phase varies from $-\pi$ to π in the image with negative phase corresponding to paramagnetic substances (iron, deoxyhemoglobin etc..) and positive phase corresponding to diamagnetic substances (calcium).

SWI - susceptibility weighted imaging: Special data processing

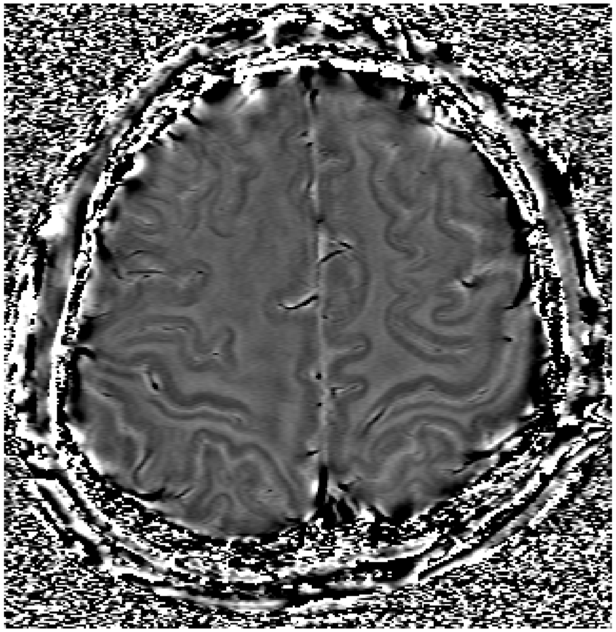
Special data processing: Use the phase information to enhance the contrast of the magnitude images. For veins or regions with iron that $f < 0$ or for calcium deposits $f > 0$.



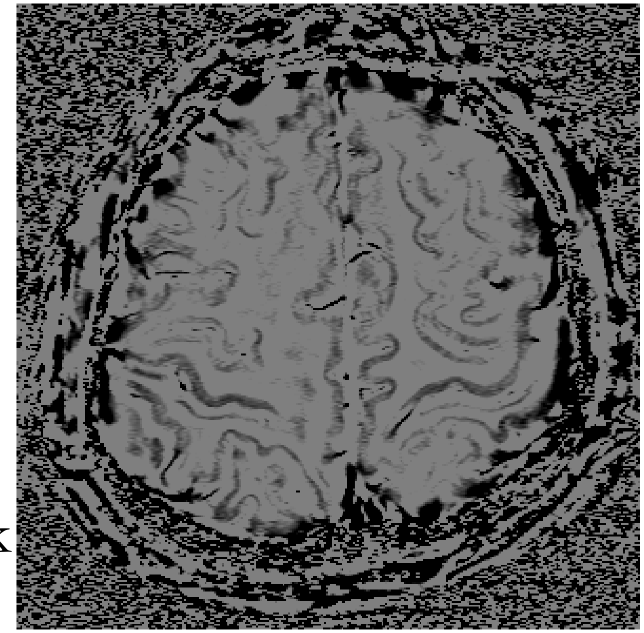
Phase Mask for enhancing paramagnetic regions

Final image,

$r_{\mathbf{SWI}} = r_{\mathbf{magnitude}}$ *** [f]^n,** where n is usually 4.

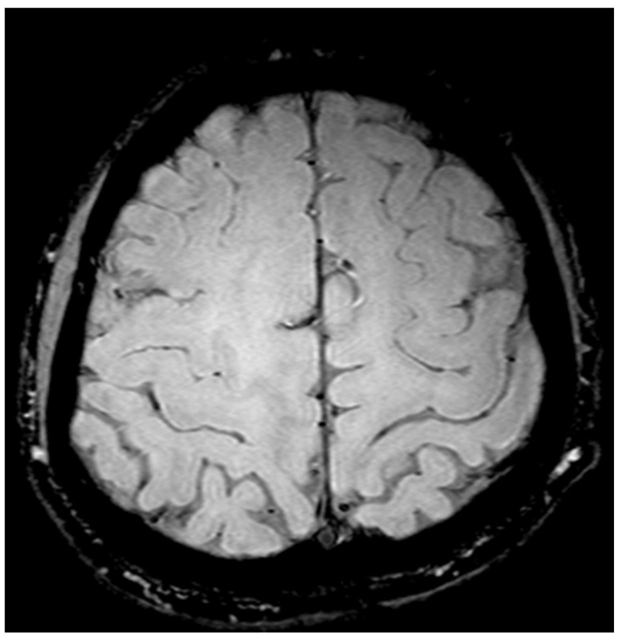


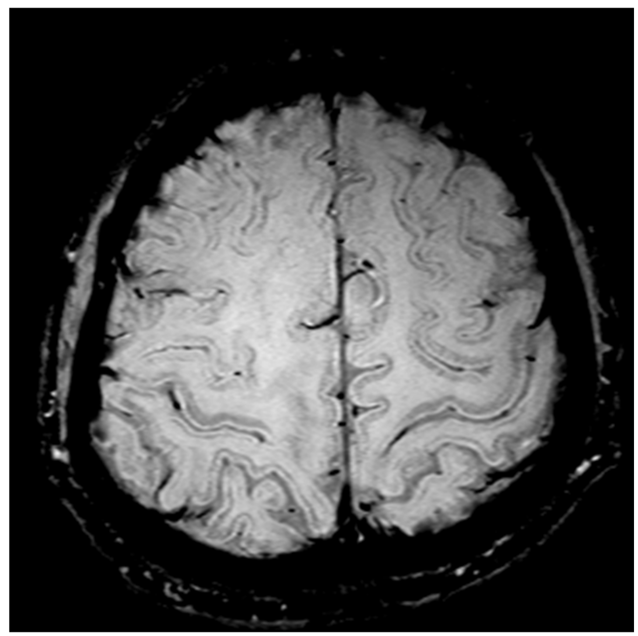
Left: Filtered Phase



Right: Mask

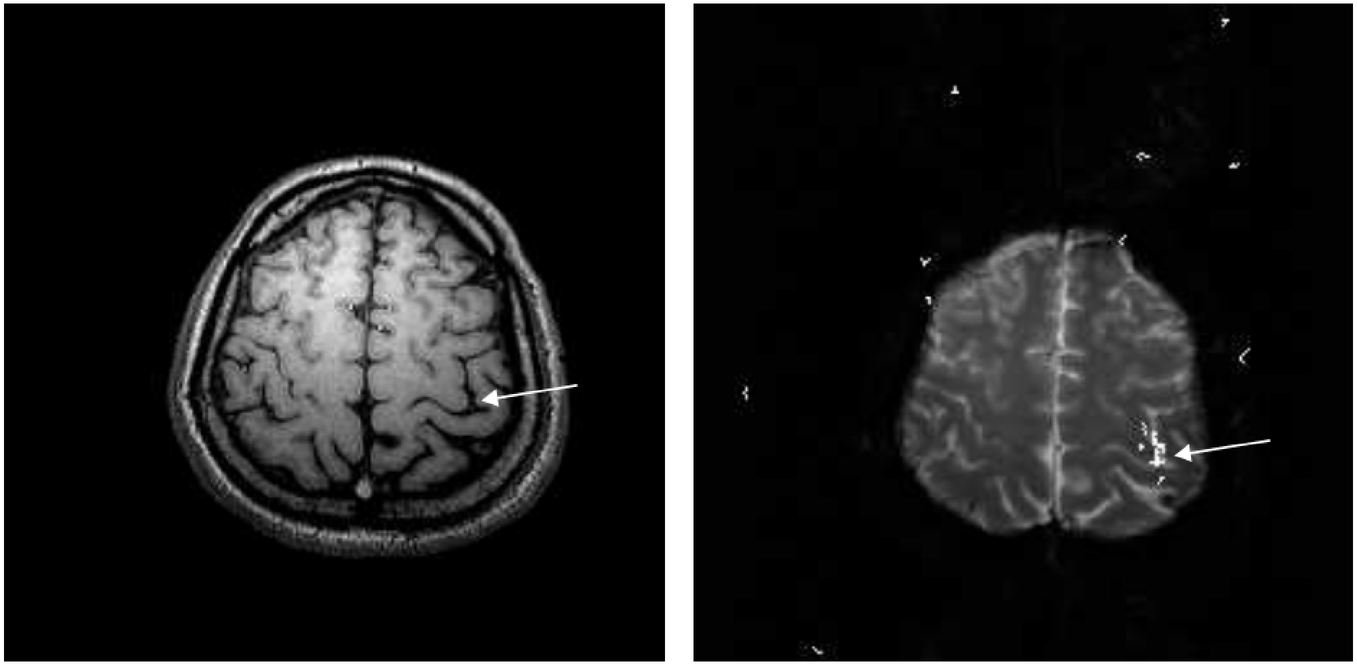
Original Magnitude





Processed Magnitude

Functional MR Imaging (fMRI)



Activated state – Inactive state